Lewis O. Harvey, Jr.–Instructor
Rosi Kaiser–Assistant
Vyga Kaufmann –Assistant
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# Psychophysical Scaling

**Fechner's Law (Fechner, 1860)**

R = unit of stimulus measurement (Reiz)
S = unit of subjective measurement
K, k, C and c are constants

(1) $\frac{\partial R}{R} = K$ (for and at the JND) E. H. Weber's Law

(2) $\partial S = c \cdot \frac{\partial R}{R}$ Fundamentalformel

Integrate to get the magnitude of S (sensation)

(3) $S = c \cdot \ln R + C$

At threshold, $S = 0$, $R = r$, therefore: $0 = c \cdot \ln r + C$ or $C = -c \cdot \ln r$
Substituting this relationship into (3):
$S = c \cdot \ln R - c \cdot \ln r$ or $S = c \cdot (\ln R - \ln r)$ or

(4) $S = c \cdot \ln\left(\frac{R}{r}\right)$

Converting Equation 4 from natural logarithms to base 10 logarithms

(5) $S = k \cdot \log\left(\frac{R}{r}\right)$ Massformel

Making $r$ (the threshold stimulus value) the unit of $R$, we have:

(6) $S = k \cdot \log R$ Fechner's Law

Conditions for the validity of Fechner's Law:

1. Unit of R is r (threshold value)
2. Integration of Equation 2 is valid
3. Valid to assume that $S = 0$ at $R = r$
4. That Weber's Law is valid

Major Criticisms

1. All JNDs do not give rise to equal sensation magnitudes
2. Sensations don't only have magnitude, but also quality (e.g., color, pitch)

**Stevens' Law (Stevens, 1936)**

*20 JND's above auditory threshold do not sound twice as loud as 10 JND's above threshold.*

Prothetic Continua

$$S = k \cdot R^n$$

$$S = k \cdot (R - r)^n$$

$$\log S = \log k + n \cdot \log R \,. \qquad \text{Stevens' Law}$$

Metathetic Continua

$$S = k \cdot \log R \qquad \text{Fechner's Law}$$

**General Psychophysical Law (Norwich & Wong, 1997)**

Both Fechner's Law and Stevens' Law can be viewed as special cases of a more general law based on information processing principles.

$$S = \tfrac{1}{2} \cdot k \cdot \ln(1 + \gamma \cdot R^n) \qquad \text{Complete Form}$$

Note that when $\gamma \cdot R^n >> 1$ the complete form becomes

$$S = \tfrac{1}{2} \cdot k \cdot n \cdot \ln R + \tfrac{1}{2} \cdot k \cdot \ln \gamma \qquad \text{Equivalent to Fechner's Law}$$

which is really Fechner's Law since $\tfrac{1}{2}kn$ and $\tfrac{1}{2}k \ln \gamma$ are constants.
The Taylor series expansion of the complete form has a first-order term of

$$S = \tfrac{1}{2} \cdot k \cdot \gamma \cdot R^n \qquad \text{Equivalent to Stevens' Law}$$

## References

Fechner, G. T. (1860). *Elemente der Psychophysik*. Leipzig, Germany: Breitkopf and Härtel.

Norwich, K. H., & Wong, W. (1997). Unification of psychophysical phenomena: The complete form of Fechner's law. *Perception & Psychophysics, 59*(6), 929–940.

Stevens, S. S. (1936). A scale for the measurement of a psychological magnitude: loudness. *Psychological Review, 43*, 405–416.