.Psychophysical Scaling

Fechner's Law (Fechner, 1860)

R = unit of stimulus measurement (Reiz)
S = unit of subjective measurement
K, k, C and c are constants

(1) $\frac{\partial R}{R} = K$ (for and at the JND) E. H. Weber's Law

(2) $\partial S = c \cdot \frac{\partial R}{R}$ Fundamentalformel

Integrate to get the magnitude of S (sensation)

(3) $S = c \cdot \ln R + C$

At threshold, $S = 0$, $R = r$, therefore: $0 = c \cdot \ln r + C$ or $C = -c \cdot \ln r$
Substituting this relationship into (3):
$S = c \cdot \ln R - c \cdot \ln r$ or $S = c \cdot (\ln R - \ln r)$ or

(4) $S = c \cdot \ln\left(\frac{R}{r}\right)$

Converting Equation 4 from natural logarithms to base 10 logarithms

(5) $S = k \cdot \log\left(\frac{R}{r}\right)$ Massformel

Making r (the threshold stimulus value) the unit of R, we have:

(6) $S = k \cdot \log R$ Fechner's Law

Conditions for the validity of Fechner's Law:

1. Unit of R is r (threshold value)
2. Integration of Equation 2 is valid
3. Valid to assume that $S = 0$ at $R = r$
4. That Weber's Law is valid

Major Criticisms

1. All JNDs do not give rise to equal sensation magnitudes
2. Sensations don't only have magnitude, but also quality (e.g., color, pitch)

Stevens' Law (Stevens, 1936)

20 JND's above auditory threshold do not sound twice as loud as 10 JND's above threshold.

Prothetic Continua

$$S = k \cdot R^n$$

$$S = k \cdot (R - r)^n$$

$$\log S = \log k + n \cdot \log R. \qquad \text{Stevens' Law}$$

Metathetic Continua

$$S = k \cdot \log R \qquad \text{Fechner's Law}$$

General Psychophysical Law (Norwich & Wong, 1997)

Both Fechner's Law and Stevens' Law can be viewed as special cases of a more general law based on information processing principles.

$$S = \tfrac{1}{2} \cdot k \cdot \ln(1 + \gamma \cdot R^n) \qquad \text{Complete Form}$$

Note that when $\gamma \cdot R^n >> 1$ the complete form becomes

$$S = \tfrac{1}{2} \cdot k \cdot n \cdot \ln R + \tfrac{1}{2} \cdot k \cdot \ln \gamma \qquad \text{Equivalent to Fechner's Law}$$

which is really Fechner's Law since $\tfrac{1}{2}kn$ and $\tfrac{1}{2}k\ln\gamma$ are constants.
The Taylor series expansion of the complete form has a first-order term of

$$S = \tfrac{1}{2} \cdot k \cdot \gamma \cdot R^n \qquad \text{Equivalent to Stevens' Law}$$

References

Fechner, G. T. (1860). *Elemente der Psychophysik*. Leipzig, Germany: Breitkopf and Härtel.

Norwich, K. H., & Wong, W. (1997). Unification of psychophysical phenomena: The complete form of Fechner's law. *Perception & Psychophysics, 59*(6), 929–940.

Stevens, S. S. (1936). A scale for the measurement of a psychological magnitude: loudness. *Psychological Review, 43*, 405–416.